Black Hole Thermodynamics: A Literature Review

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Abstract

In this paper, the topics of black hole thermodynamics are discussed. It begins with an overview on the subject, bringing emphasis on what exactly black holes are in a general sense and how they're mathematically expressed as. The topics of thermodynamics aren't talked about in detail as it assumes the reader is very familiar with this topic. Next, the paper goes into discussion about the history and background of black hole thermodynamics, like the impact of Hawking radiation on the development and Bekenstein's proposal of black hole entropy. Then, the introduction to the four "thermodynamic" black hole laws are introduced. This is the biggest discussion topic of the paper, and more emphasis will be on this section. The relation of black hole "temperature" is brought in, with helping to make the connection with the four thermodynamic laws more rigid in its connection with black hole thermodynamics. Lastly, it dives into the astronomical implication of black hole thermodynamics and how research may process in the future on the topic in physics.

Introduction

Black holes are such a phenomena in the universe that there's been over 240 years of them being first proposed thanks to John Michell, an English Parson [6]. Black holes can be defined as intense regions of gravity in space from which nothing, not even light, can escape [4]. They all vary in size, where one the largest black holes is Abell 1201, which has a mass of 30 billion suns [5], and the smallest, V723 Mon, also called "The Unicorn", has a mass about the size of the Sun [7]. All black holes are defined with an event horizon, which is essentially the radius at which the coined phrase "not even the speed of light can escape". One can consider the inside of this region to require an escape velocity,

$$v_{esc} > c \tag{1}$$

where,

$$c = 299,792,458 m/s \tag{2}$$

also known as the speed of light in a vacuum, which was proposed by Einstein. Einstein's paper, "On the Electrodynamics of Moving Bodies", proposed the foundation and ground work for the entire field of relativity, where he introduced the speed of light. Going back go black holes, there is actually a correction that's needed to be in place to correctly explain how they work. In correct terminology, it's actually the *curvature of space*, not the gravity itself, that causes light to not be able to escape. Space time is the biggest player in Relativity, where whether or not it's curved differentiates general and special relativity. Mass of any size bends space time. Black holes are proposed to be very dense objects, where the curvature of space time is so great that it forms at a singularity, where the normal laws of physics don't apply there because space time breaks down so much at that point.

Figure 1 shows that the curvature for a black hole is essentially infinite. It requires a lot of energy in order to move out of an event horizon. In general relativity, non-rotation black holes can be described by the Schwartzchild metric:

$$ds^2 = -c^2 d\tau^2 = -(1 - \frac{r_2}{r})c^2 dt^2 + \frac{1}{1 - \frac{r_s}{r}} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3}$$

where r_s is the Schwartzchild radius. The Schwartzchild radius is the radius that an object must be dependent on mass in order to turn into a black hole. This is represented by

$$r_s = \frac{2GM}{c^2} \tag{4}$$

The Schwartzchild radius of earth is $8.87 \times 10^{-3} m$. If you crushed the Earth into this small of a radius, then you could attain a black hole.

The importance in black holes thermodynamics is to find how black holes work, and how they interact with masses that move in. By doing this, we can soon predict how black holes will change, as well as open the insight of the connection to thermodynamics.Thermodynamics is also a topic that is talked more heavily in life, with many classes that go into the origins of thermodynamics. I won't go into detail about those, but the topics of most importance lies in the 4 laws of thermodynamics, which are:

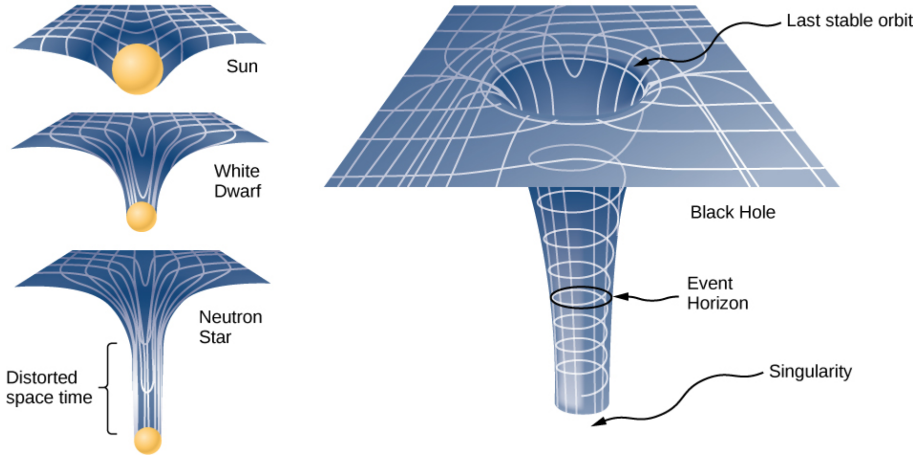

Figure 1: A visualization of space time [1]

- Zeroth law:
  - If two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.
- First law:
  - Energy cannot be created nor destroyed, only conserved.
- Second law:
  - The entropy of any isolated system always increases over time.
  - Heat cannot spontaneously flow from a colder object to a hotter object.
- Third law:
  - As temperature decreases, so does entropy at equilibrium, and absolute 0 cannot be achieved

These will play a key role in associating with black holes. Why? Because we can actually relate black holes to temperature.

Fundamentals of Black Hole Thermodynamics

Black holes are considered to be in equilibrium when they are stationary and non-rotating. This means that gravity is uniform at the event horizon. This is completely analogous to the zeroth law of thermodynamics in the sense of equilibrium.

Before we can go further into the thermodynamics of black holes, we need to touch on an important topic: entropy. Entropy can be regarded as a measure of disorderliness. It's apart of the second and third law of thermodynamics. An isolated system (a system that is in the sense "cropped" out and only focused on) can *never* decrease in entropy, only increase. This is because in order to decrease entropy, you would need to destroy it. Entropy can be considered as a system's thermal energy that is unavailable to perform work. This means that in a sense entropy is energy, so we obviously cannot destroy energy thanks to the first law of thermodynamics. Entropy "holds information" about a system. You cannot destroy information. When you approach absolute zero, your entropy will be at a minimum.

Black holes can be characterized by their mass M, charge Q, and angular momentum J. This is also known as the No-hair Theorem [2]. With the No-Hair Theorem, we can simply an identity relating the changes regarding these parameters for black hole when it is perturbed. We can do so with

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega\delta J \tag{5}$$

Equation (5) is actually known as the first law of black hole thermodynamics where κ is the surface gravity of a black hole and is given by

$$\kappa = lim(Va) \tag{6}$$

where $V \equiv \sqrt{-\xi^a\xi_a}$ is the Redshift factor of ξ^a, which ξ^a is a killing field. The Redshift factor is the limit as one approaches $\mathcal{K}$, where $\mathcal{K}$ "whose null generators coincide with the orbits of a one-parameter group of isometries [which is a map where the distance between objects are the same distance] [8]". What the first law of black hole thermodynamics states is that all 3 of these quantities must be conserved, similar to the first law of thermodynamics. These quantities cannot be created nor destroyed.

Black holes have no entropy, so with this theorem, the only entropy that is being concerned for is the object or matter that is interacting with the black hole. What's so special about this? This means that everything else about the black hole is lost when it enters a black hole. There is a big issue with this: we just lost information! This is a violation of the second "thermodynamic law" of black holes. In order to counter-react this, Jack Beckenstein proposed that black holes actually have entropy! The entropy of a black hole is given by

$$S_{BH} = \frac{k_B A}{4\ell_p^2} \tag{7}$$

where k_B is the Boltzmann constant $(1.380649 \times 10^{-23} m^2 kg s^{-2} K^-1$, A is the surface area of the black hole, and ℓ_p is the Plank length $(1.616255 \times 10^{-35} m.$ However, this only applies to a stationary, non-rotating black hole. Now that we have this expression for the entropy of a black hole, we are now not violating any laws of thermodynamics with the loss of entropy. Equation (7) is known as the Bekenstein-Hawking entropy. When you combine 2 black holes, the surface area of the new black hole is larger than the surface area of the previous black holes. This is the second law of black hole thermodynamics.

If we want to relate the concept of temperature with black holes, then we need to talk about Hawking radiation. Due to quantum particle creation infinitely radiates with perfect black body radiation at the temperature

$$T = \frac{\kappa}{2\pi} \tag{8}$$

This is a good representation of a physical temperature of a black hole. "Physical" in the sense is that there's not a ambiguous mathematical construct or theory with this; we can actually think and see this [8]. This equation is known as the Hawking effect. this is proportional to the surface gravity of a black hole.

If we want to discuss the third law of black hole thermodynamics, then we should start with an educated guess on how that would look. It would look like "As $S \to 0$, then $T \to 0$. We've already discussed the entropy of a black hole, so this will not work out. What we can manipulate this to is the fact that the surface gravity cannot be 0. We can get extremely close, but we can never get to 0. The actual temperature of a standard black hole is $1.4 \times 10^{-14} K$. Look at that magnitude!

Historical Development of Black Hole Thermodynamics

Bekenstein's proposal for black hole entropy in the 1970's was based off of the original idea that black holes have entropy that is proportional to their surface area rather than volume. This was a major breakthrough in the study of black holes. Obviously when you propose an idea that is really out there, not many people are going to agree with your right off the bat. That's why the contribution of Hawking radiation really helped out with this.

Hawking Radiation

When matter enters black holes, we don't know what happens when they enter them, but what we do know is that we lose information. When an observer is near the event horizon, they can see the particles moving if the observer is moving, but they're stationary if the observer is not moving. This is known as the Unruh Effect [9]. This thermal bath of particles is important because due to the absence of heat, you still have a set of moving particles. An observer falling into an event horizon will be in an inertial frame and see these moving particles and be heated up. There's a paradox that falls in line with this, where the moving observer will start to heat up due to the friction of the particles, but to a stationary observer, they'd be confused to why the moving observer just randomly caught on fire (obviously the would not catch on fire due to lack of oxygen, but they'd still burn up). These black holes undergo perfect black body emission of many atomic particles. This is due to matter and antimatter. Because the two particles are tied together, when brought in contact with an event horizon, the antimatter is sucked in whereas the matter escapes. The antimatter actually has negative energy, so it will decrease the energy of the black hole. This means that the more particles it sucks, the less mass energy it will have.

Figure 2 shows a visualization of Hawking radiation. As particles are ripped apart, there is a clear exchange with the absorption and release of particles.

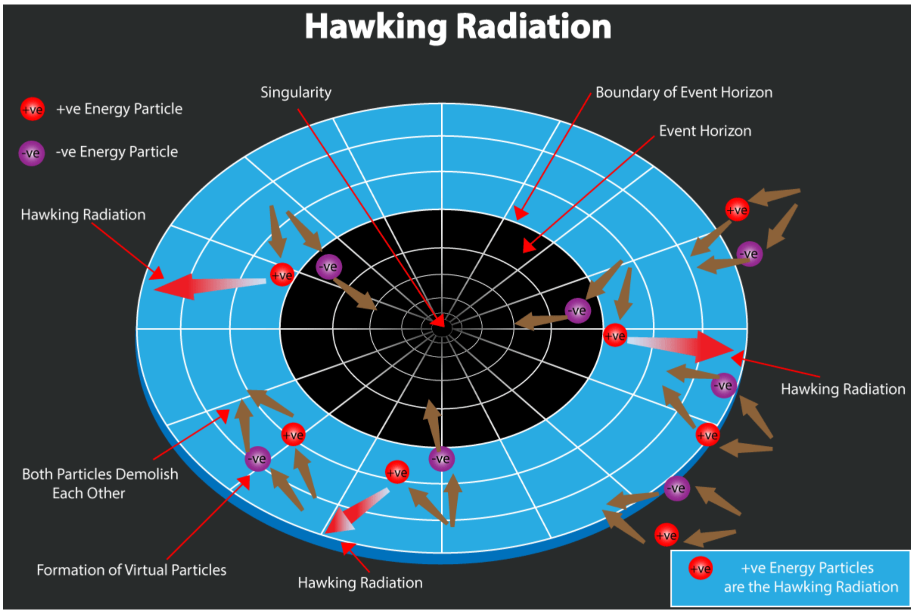

Figure 2: A visualization of the particle interaction within the area of an event horizon and the exchange of matter. [3]

Astronomical Implications of Black Hole Thermodynamics

The main method that goes into our research about black holes is to not solely rely on the thermodynamics of black holes. What we want to is to use our methodology of thermodynamics so that we can pretend that we're doing thermodynamics with black holes. Our big focus is on losing information. When we introduced Hawking radiation, we ran into the information paradox. We still need to perform more work on the information loss that goes into matter going inside of black holes.

One big genre to focus on is the connection is with quantum mechanics and general relativity. This is because of the size difference regarding the particles of both theorems. Quantum mechanics has to deal with very small objects, whereas general relativity reals with very large objects. the curvature for an electron would be so incredibly low for something of that size. The Schwartzchild radius of an electron would be around $1.35 \times 10^{-57}m$, which is 8.35×10^{-23}. No word in the dictionary could possibly describe the length of that, or even fathom any kind of comparison that wouldn't not make the audience question reality. There is a lot of work that still needs to be done.

What's next?

There are still many gaps in our understanding of black hole thermodynamics. To start: We still don't have a 100% understanding about entropy. It's a very hand-wavy topic in thermodynamics. We still don't know about the information paradox, to what down-to-the-nitty-gritty happens and why. We also don't know about the connection with quantum mechanics. This would fall into the lines of statistical mechanics. Lastly, there's the fact about actually looking at black holes. It was already difficult in the first place to observe the black holes, but actually performing studies and analysis on them? There's no way that we can do that in our current age. These are the most important pillars in developing black hole thermodynamics so that we can better understand what's going on. Once we do that, then we can understand black holes a lot easier.

Conclusion

Black hole thermodynamics is a very advanced topic but an important topic. It's the key for helping us to be able to understand black holes. By connecting it with a topic that we already know so much about and put in so much work, then we can try and piece things together. By using the connection of entropy, then we are able to find an easier connection to thermodynamics. The universe is governed by entropy and energy. On the basis of conservation, we are at will of how to analyze certain topics. We must make sure to never destroy or create things without worrying about making sure that certain quantities are conserved. From particles to information, studying black hole interaction must be understood by first relating topics to thermodynamics. One day, we will be very and efficiently qualified to talk about black holes.

Acknowledgments

I want to give a big thanks to Dr. Dimitrios Psaltis for distributing [8] to me so that I may use it for research on this topic. The study of thermodynamics is already an interesting topic, but bringing it in lines with relativity makes it an even more interesting subject.

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